

Answer Key

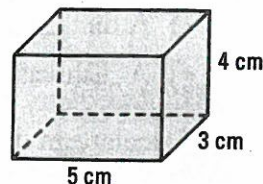
ACTIVITY 3 Volume of a prism

$$1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} = 1 \text{ cm}^3$$

- a) 1. What is the total number of cubes with 1 cm edges that the prism on the right can contain?

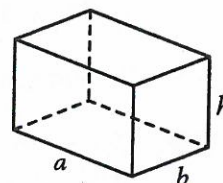
$$5 \times 4 \times 3 = 60 \text{ cubes}$$

2. What is the volume of this prism? 60 cm^3



- b) The prism on the right has a base with dimensions a and b , and h as the height. What is the volume of this prism?

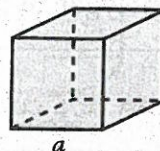
$$a \times b \times h$$



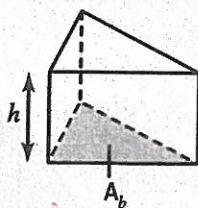
VOLUME OF A PRISM

- The volume of a cube with edge length a is:

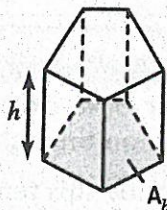
$$V = a \times a \times a = a^3$$



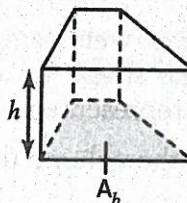
- The volume of a prism is equal to the product of the area of the base A_b and the height h of the prism.



$$A_b = \frac{b \times h}{2}$$



$$V = A_b \times h$$

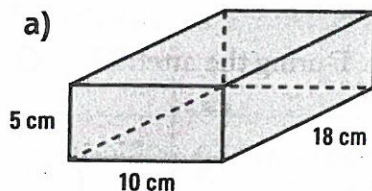


Area of any regular polygon =

$$= \frac{(\text{Apothem} \times \text{perimeter})}{2}$$

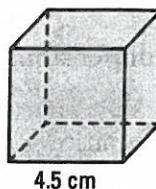
17. Calculate the volume of each of the following prisms.

a)



$$5 \times 10 \times 18 = 900 \text{ cm}^3$$

b)

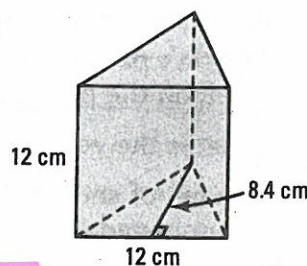


$$4.5^3 = 91.125 \text{ cm}^3$$

or

$$4.5 \times 4.5 \times 4.5$$

c)



$$V = A_b \times h$$

$$= \left(\frac{b \times h}{2} \right) h$$

$$= (50.4) 12 = 604.8$$

$$193 \text{ cm}^3$$

Many different answers for #18

Varied Answers

18. Estimate, using the appropriate units of volume, the volume of the following prisms.

- a) A \$1 coin mm^3 or cm^3 b) Your classroom m^3
 c) A die cm^3 d) Your math book cm^3 or dm^3
 e) A dishwasher dm^3 f) Your house m^3

19. Calculate, in dm^3 , the volume of a prism with the dimensions 4.5 dm, 0.3 m and 40 cm. 54 dm^3
 $\times 3 \text{ dm} \times 4 \text{ dm}$
 $\div 10 = 4 \text{ dm}$

20. a) Calculate, in m^3 , the volume of a cube with 40 dm edges. 64 m^3
 $4 \times 4 \times 4 = 64$

b) How many times does the volume increase if the edge length doubles? 8 times
 $8 \times 8 \times 8 = 512 \div 64 = 8 \text{ times}$

21. Express, in litres, the volume of a cube with 11 cm edges. $1331 \text{ mL} = 1.331 \text{ L}$
 $11 \times 11 \times 11 = 1331 \text{ cm}^3 = 1331 \text{ mL}$

22. The mass of 1 l of air is 1.3 g. What is the mass, in kg, of the air in a classroom with a length of 12 m, a width of 8 m and a height of 5 m? 624 kg
 $480 \text{ m}^3 = 480000 \text{ dm}^3 \times 1.3 \text{ g} = 624000 \text{ g}$
 $(\text{L}) = 624 \text{ kg}$

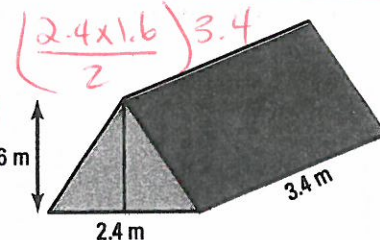
23. Calculate the volume of a \$1 coin knowing that it has the shape of a right prism with an 11-sided regular polygon as its base with sides of 0.7 cm, an apothem of 1.2 cm and a height of 2 mm. 0.924 cm^3
 $P = (11 \times 0.7) = 7.7$ Area = $(1.2 \times 7.7) = 4.62 \times 0.2 = 0.924$
 0.2 cm base height

24. What is the cost of a cubic block of granite with 7 dm edges if the granite costs \$600/ m^3 ? 205.84
 $(0.7)(0.7)(0.7) = 0.343 \times 600 = 205.84$
 0.7 m

25. A driveway is rectangular. Its dimensions are 8.6 m by 4.8 m. A uniform 5 cm thick layer of asphalt is spread over this driveway. What will the cost of the asphalt be if it costs \$145/ m^3 ? 299.28
 $8.6 \times 4.8 \times 0.05 = 2.064 \text{ m}^3 \times 145 = 299.28$
 0.05 m

26. Sylvie and Fran went camping in the Laurentians. They used a tent in the shape of a triangular base prism with the dimensions represented by the figure on the right.

- a) What is the volume they have inside this tent? 6.528 m^3
 b) What is the total area of canvas used to make this tent if the floor is not made of canvas?



27. We want to store cubes with 1.5 cm edges in a box shaped like a rectangular prism with dimensions 0.75 m, 0.6 m and 4.5 m. What is the maximum number of cubes that this box can contain? 60000 cubes
 $\text{Box} = 0.75 \times 0.6 \times 4.5 = 2.025 \text{ m}^3 \div (0.015^3) = 60000$
 0.015 m 0.015 m

28. Mr. Martin's patio is rectangular with dimensions 15 m by 8 m. During the afternoon, 5 cm of snow fell on the patio.

- a) What is the volume of snow on the patio? 6 m^3
 $15 \times 8 \times 0.05 = 6$
 0.05 m
 b) If 1 m^3 of snow gives 60 l of water, what volume of water will be produced once this accumulated snow melts? 360 L
 $6 \text{ m}^3 \times 60 = 360 \text{ L}$

* The area of any regular polygon is given by the formula: Area = $\frac{\text{apothem} \times \text{perimeter}}{2}$



29. The prism on the right represents a tank filled with 1500 l of gas. How much time will be required to fill the rest of the tank at a rate of 20 litres/min?

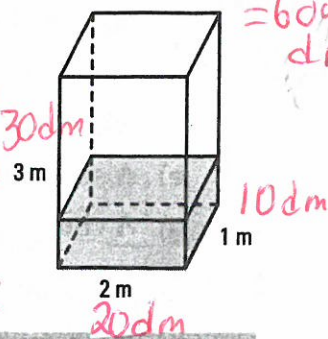
$$6000 \text{ dm}^3 = 6000 \text{ L}$$

$$6000 \text{ L} - 1500 \text{ L} = 4500 \text{ L}$$

$$4500 \text{ L} \times \frac{1 \text{ min}}{20 \text{ L}} = 225 \text{ minutes}$$

30. The dimensions of a classroom are 9 m by 6 m by 3.75 m. How many people can this classroom hold if each person must have 4.5 kl of air?

$$90 \times 60 \times 3750 = 175500 \text{ dm}^3 \div 4500 \text{ L} = 39 \text{ people}$$



ACTIVITY 4 Volume of a cylinder

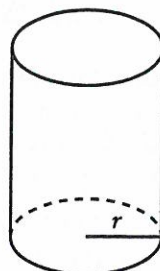
- a) Given a cylinder with radius r and height h . One can consider, taken to the extreme, this cylinder as a right prism with the same height in which the base is a regular polygon with a very large number n of sides.

1. What is the area of the cylinder's base?

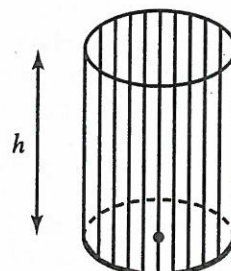
$$\pi r^2$$

2. Give a formula for calculating the volume of the cylinder.

$$V = A_b \times h = \pi r^2 h$$



cylinder



prism

- b) Determine the volume of a cylinder with a 4 cm radius and a height of 6 cm.

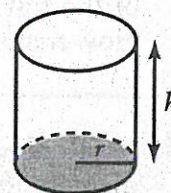
$$V = 96\pi \text{ cm}^3 = 301.8 \text{ cm}^3$$

VOLUME OF A CYLINDER

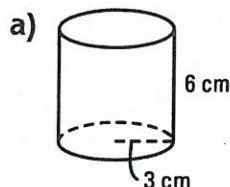
The volume of a right circular cylinder, with radius r and height h , is equal to the product of the area of the cylinder's base A_b and the height h of the cylinder.

$$V = A_b \times h = \pi r^2 h$$

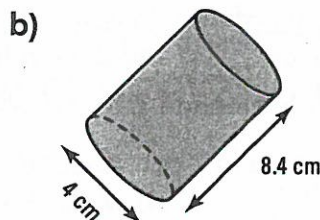
$$A_b = \pi r^2$$



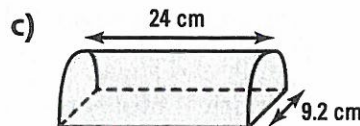
31. Calculate the volume of each of the following cylinders and half-cylinder.



$$54\pi \text{ cm}^3$$



$$33.6\pi \text{ cm}^3$$



$$253.92\pi \text{ cm}^3$$

① $V = \pi r^2 h$

$$= \pi 3^2 6 = 54\pi$$

32. Estimate, using the most appropriate unit of volume, the volume of the following cylinders.

- a) An above-ground pool L b) A tea cup cm³
 c) A quarter mm³ d) A white-out bottle mL
 e) A soft drink can mL f) A tin can dm³ or cm³
 g) A cheesecake dm³ h) A new pencil cm³

- 33.** a) Calculate the volume of a cylinder with a 4 cm radius and a height of 6 cm. $96\pi \text{ cm}^3$
 b) If we double the cylinder's height, what happens to the volume? it also doubles
 c) If we double the cylinder's radius, what happens to the volume? it quadruples

34. Express, in litres, the volume of a cylindrical container with a 7 dm diameter and a height of 20 cm. $\pi (3.5)^2 (2) = 76.97 \text{ L}$

35. A small cylindrical shaped bottle of perfume is sold for \$48 (taxes included). If the radius of the bottle is 3.5 cm and its height is 10 cm, how much does the perfume cost per litre?

$\pi (3.5)^2 (1) = 0.38 \text{ dm}^3$ $\frac{48}{0.38} = 124.72 \$$

36. The glass top of a circular table has a diameter of 1.4 m and a thickness of 1 cm. Knowing that 1 dm³ of glass has a mass of 2 kg, calculate the mass of the glass top of the table.

$\pi r^2 h = \pi (7)^2 (0.1) = 15.39 \text{ dm}^3 \times 2 \text{ kg} = 30.79 \text{ kg}$

37. A cylindrical tunnel with a 21 m diameter is dug over a distance of 160 m. How many trips must a truck carrying 45 m³ of dirt make to carry away all of the dirt that was dug?

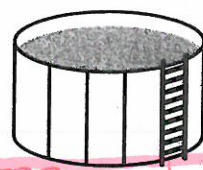
$\pi r^2 h = \pi (10.5)^2 (160) = 55417.69 \text{ m}^3 \div 45 \text{ m}^3 = 1232$

38. A pot has the shape of a cylinder with a 28 cm diameter and a height of 15 cm. The contents of the full pot are emptied into cylindrical cups with a 7 cm diameter and a height of 3 cm. How many cups can be filled to capacity?

$\pi r^2 h = \pi (14)^2 (15) = 9236.28 \text{ cm}^3$ $\pi (3.5)^2 (3) = 115.45 \text{ cm}^3$
 $79.99 = 80 \text{ cups}$

39. Mr. Reid wants to fill the above-ground pool in his backyard. The pool has a diameter of 4 m and a height of 1.8 m. If he fills the pool to $\frac{3}{4}$ capacity, how long will it take to fill it at a rate of 45 l/min?

$\pi (2)^2 (1.8) = 22619.47 \text{ dm}^3 \times \frac{3}{4} = 16964.60 \times \frac{1 \text{ min}}{45} = 377 \text{ minutes}$



40. Mrs. Carter wants to make muffins for her guests. She prepares 2 litres of muffin mix with eggs, milk and sugar. What quantity of mix, in cm³, will be left over if she pours the mix into 18 cylindrical molds with a 7 cm diameter and 2.5 cm height?

$\pi (3.5)^2 (2.5) = 96.21 \text{ cm}^3 \times 18 = 1731.80 \text{ cm}^3$ $2000 - 1731.80 = 268.20 \text{ cm}^3$

41. A triangular base prism is submerged in a cylindrical bucket of water with a 5 cm radius. The water level in the bucket rises 0.8 cm. What is the volume of the prism submerged in the water?

This is a water displacement question
→ remember in science?
 $\text{rises} = 0.8 \text{ cm} = \text{volume raised} = \pi (5)^2 (0.8)$
 $= 62.83 \text{ cm}^3 = \text{volume of prism}$



42. Given the rectangle on the right.

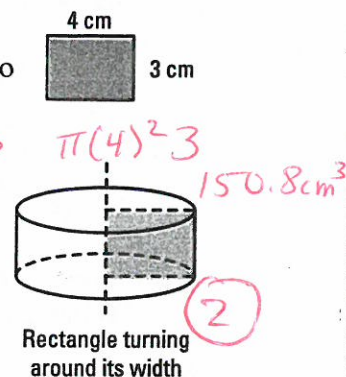
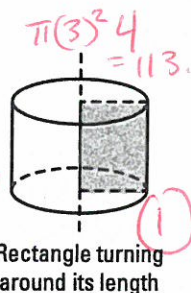
This rectangle is rotated around its length and then around its width. The two cylinders on the right are therefore generated.

a) Do the two cylinders have the same volume?

Justify your answer.

No because the radius's height are different

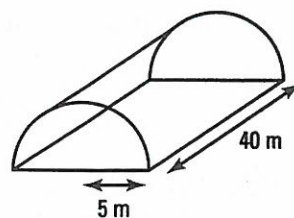
b) Calculate the volume of each cylinder.



43. Mr. Denton grows plants in a semi-cylindrical greenhouse with a 5 m radius and a height of 40 m. If each plant requires 0.75 m^3 of air, how many plants can grow in this greenhouse?

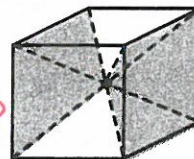
$$(\pi r^2 h) \div 2 \rightarrow (\pi (5)^2 (40)) \div 2 = 1570.8 \text{ m}^3$$

$$1570.8 \div 0.75 = 2094 \text{ plants}$$



ACTIVITY 5 Volume of a pyramid

Given the cube on the right with edge length a that is separated into 6 identical pyramids, each using a cube's face as its base and half of the cube's edge length as its height.



a) What is the volume of each pyramid?

$$\frac{a^3}{6}$$

b) One of the formed pyramids is represented here on the right.

1. What is its height?

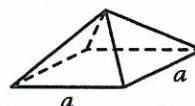
$$\frac{a}{2}$$

2. What is the area of the base?

$$a^2$$

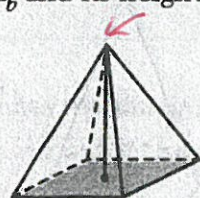
3. Verify that the volume V of the pyramid is $V = \frac{A_{\text{base}} \times \text{height}}{3}$.

$$\frac{a^3}{6} = (a^2 \cdot \frac{a}{2}) \div 3$$



VOLUME OF A PYRAMID

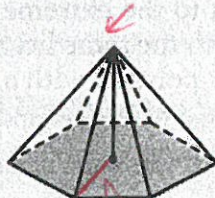
The volume of a right regular pyramid is equal to the third of the product of the area of the pyramid's base A_b and its height h .



square



triangle



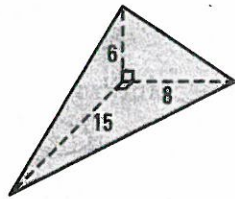
hexagon an apothem

$$V = \frac{A_b \times h}{3}$$

This formula also enables you to calculate the volume of non-regular pyramids.

- 44.** Calculate the volume of each of the following pyramids (the measures are given in centimetres).

a)



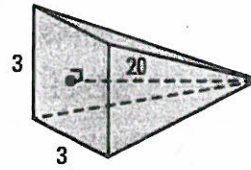
$$V = \frac{A_b \times h}{3}$$

$$= \frac{(8 \times 6) \cdot 15}{3}$$

$$= \frac{720}{3}$$

$$= 240 \text{ cm}^3$$

b)



$$= \frac{(3 \times 3) \cdot 20}{3}$$

$$= 60 \text{ cm}^3$$

- 45.** The height of a pyramid is 156 mm. The base is a rectangle that is 24 cm long and 12 cm wide. What is the volume of this pyramid?

- 46.** The base of a pyramid is a right triangle with its sides measuring 2.4 cm, 3.2 cm and 4 cm. The height of this pyramid is 8 cm. Calculate its volume.

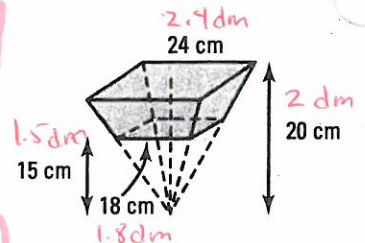
- 47.** A square base pyramid with 12 cm sides has a slant height of 10 cm. What is the volume of this pyramid?

- 48.** The base of a pyramid is a square with 12 cm sides. The lateral edge measures 10 cm. Calculate this pyramid's volume.

- 49.** A trophy has the shape of a square base pyramid. The height of the pyramid is 6 cm and its slant height is 10 cm. If the trophy is made of aluminum and the mass of 1 dm³ of aluminum is 2.7 kg, calculate the mass of this trophy.

- 50.** Calculate the approximate volume of the Cheops pyramid in Egypt knowing that it has a square base with sides of approximately 230 m and a slant height of almost 180 m.

- 51.** A flower pot has the shape represented on the right. What is the volume (in litres) of dirt that this pot contains when it is filled to the top?



ACTIVITY 6 Volume of a cone

Given a cone with height h and a base of radius r . One can consider, taken to the extreme, this cone as a right pyramid with the same height in which the base is a regular polygon with a very large number n of sides.

- a) 1. What is the area of the cone's base?

$$\pi r^2$$

2. Find a formula for the volume of a cone.

$$V = \frac{A_b \times h}{3} = \frac{\pi r^2 h}{3}$$

- b) Determine the volume of a cone with a height of 10 cm and having a disc with a 6 cm radius as its base.

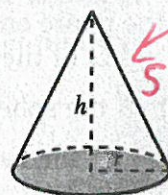
$$\frac{\pi (6)^2 \cdot 10}{3} = 120\pi \text{ cm}^3$$

VOLUME OF A CONE

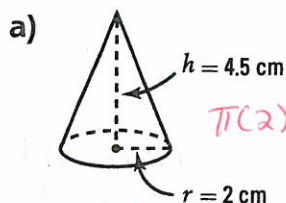
The volume of a cone with radius r and height h is equal to a third of the product of the area of the cone's base A_b and its height h .

$$V = \frac{A_b \times h}{3} = \frac{\pi r^2 h}{3}$$

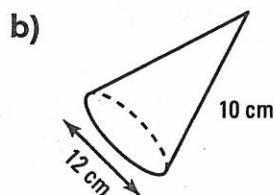
$$A_b = \pi r^2$$



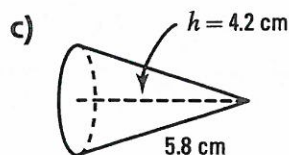
52. Calculate the volume of each of the following cones.



$$6\pi \text{ cm}^3$$



$$96\pi \text{ cm}^3$$



$$22.4\pi \text{ cm}^3$$

53. A cone has a height of 2.4 m. The cone's radius is 1.8 m. What is this cone's volume?

$$\pi(1.8)^2(2.4) = 8.14 \text{ m}^3$$

54. What is the volume of a cone with a slant height of 5.4 m and a diameter of 3.8 m? Round your answer to the nearest tenth.

$$19.1 \text{ m}^3$$

$$\frac{\pi(1.9)^2(5.05)}{3} = 1.9$$

55. A cone has a height equal to its radius. What is the volume of this cone knowing that its diameter is equal to 4.36 m?

$$\frac{\pi(2.18)^2(2.18)}{3} = 10.85 \text{ m}^3$$

56. A water cooler has a cylindrical shape with a radius of 15 cm and a height of 42 cm. It is filled to capacity. How many cups in the shape of a cone can be filled fully by this water cooler if each cup has a 3 cm radius and 7 cm height?

$$\frac{\pi(3)^2(7)}{3} = 65.98 \text{ cm}^3$$

$$\pi(15)^2(42) = 29688.1 \text{ cm}^3$$

$$\frac{29688.1}{65.98} = 450 \text{ cups}$$

57. At a reception, champagne was served in cone-shaped glasses. The radius of the cone's base is 2.7 cm and the height of the cone is 10 cm. How many glasses filled to $\frac{4}{5}$ of their height can be served from a bottle containing 0.76 l?

$$\frac{\pi r^2 h}{3} \times \frac{4}{5} = 61.07$$

$$\frac{0.76}{0.061} = 12 \text{ cups}$$

58. A plastic cup has the shape illustrated on the right. If Eric fills it to $\frac{3}{4}$ the capacity with lemonade, how much lemonade (in cl) will be poured into the cup?

Big cone: $\frac{\pi(3.5)^2(12.6)}{3} = 161.63 \text{ cm}^3$

Small cone: $\frac{\pi(1)^2(3.6)}{3} = 3.77 \text{ cm}^3$

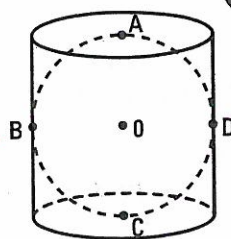
$$161.63 - 3.77 \text{ cm}^3 = 157.86 \text{ cm}^3 \times \frac{3}{4} = 118.4 \text{ cm}^3 = 118.4 \text{ cl}$$



↑ not part of the cup

ACTIVITY 7 Volume of a sphere

In the figure on the right, the sphere is inscribed in the cylinder. The points A, B, C and D of the sphere are also points on the cylinder.



a) If the sphere has a radius of r , what is

1. the radius of the cylinder? r 2. the height of the cylinder? $2r$

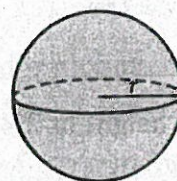
b) It has been proven that the volume of a sphere is equal to $\frac{2}{3}$ of the volume of the cylinder circumscribing it. Therefore, what is the volume of a sphere with radius r ?

$$V = \frac{2}{3}(\pi r^2 \cdot 2r) = \frac{4}{3}\pi r^3$$

VOLUME OF A SPHERE

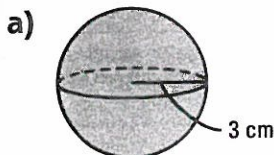
The volume of a sphere with radius r is equal to:

$$V = \frac{4\pi r^3}{3}$$



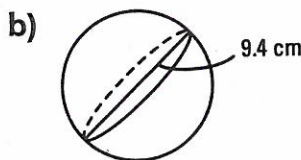
hemisphere
= $\frac{1}{2}$ sphere

59. Calculate the volume of each of the following spheres and the hemisphere.

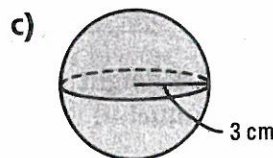


$$\frac{4\pi 3^3}{3}$$

$$36\pi \text{ cm}^3$$



$$138.43\pi \text{ cm}^3$$



$$0.52\pi \text{ m}^3$$

60. Calculate the volume of the sphere whose radius measures

- a) 4.5 cm. $121.5\pi \text{ cm}^3$ b) 2.28 m. $15.8\pi \text{ m}^3$ c) 0.9 mm. $0.972\pi \text{ mm}^3$

61. Two marbles have respective radii of 1.5 cm and 3 cm. What is the ratio of

- a) their radii? 2 b) their areas? 4 c) their volumes? 8

62. Calculate the volume of a tennis ball with a 6.5 cm diameter. 143.79 cm^3

63. Calculate the volume of half an orange with a 4 cm radius. 134.04 cm^3

64. A bowl has the shape of a hemisphere with a 0.9 m diameter. What is the capacity, in litres, of this bowl? 190.85 L

65. a) Calculate the approximate volume of planet Earth if its radius is estimated at 6400 km. 1.098×10^{12}

- b) How many times bigger is Earth compared to the Moon if the Moon's radius is estimated at 3500 km? 6 times

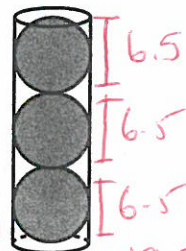
66. What quantity of air, in litres, is required to inflate a balloon with a 30 cm diameter? $\frac{4\pi r^3}{3}$ 3dm = 14.14 L

67. a) Calculate the volume of a sphere with a 3 cm radius. 113.1 cm^3

b) If the radius of this sphere is doubled, by what number does the volume get multiplied?

by 8

68. A cylindrical can contains 3 tennis balls with a radius of 3.25 cm as illustrated on the right. What is the volume of the unoccupied space in this cylinder?



$$\left(\frac{4\pi (3.25)^3}{3} \times 3 \right) = \text{space occupied by balls}$$

69. An ice cream container has the shape of a rectangular prism with dimensions 20 cm, 12 cm and 9 cm. It is filled to capacity. What is the approximate number of spherical scoops of ice cream that can be made if the diameter of one scoop is 6 cm?

$$20 \times 12 \times 9 = 2160 \text{ cm}^3$$

$$\frac{4\pi (3 \text{ cm})^3}{3} = 113.1 \text{ cm}^3$$

$$2160 \div 113.1$$

$$= 19 \text{ scoops}$$

70. How many oranges with a 6 cm diameter must be squeezed to completely fill a cylindrical can with a 6 cm radius and a height of 20 cm? (Each orange gives approximately 75% of its volume in juice.)

$$\pi (6)^2 20 = 2261.95 \text{ cm}^3$$

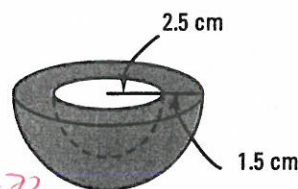
$$\frac{4\pi (3)^3}{3} = 113.1 \text{ cm}^3 \times 0.75 = 84.82 \text{ cm}^3$$

$$2261.95 \div 84.82 = 27 \text{ oranges}$$

71. The dimensions of a rectangular box are 30 cm by 25 cm by 8 cm. The following crystal solids are stored in the box: 6 spheres with a 5 cm radius; 6 cylinders with an 8 cm diameter and a 4 cm height; 10 circular cones with a 3 cm radius and a 5 cm height and 4 cubes with 3 cm edges. What is the volume of unoccupied space in this box?

$$1072.8 \text{ cm}^3$$

72. Consider the half of cantaloupe illustrated on the right. The part that has been removed corresponds to a hemisphere. What is the volume of the remaining fruit contained in this half-cantaloupe? Ignore the thickness of the peel.



$$\frac{4\pi (2.5)^3}{3} = 65.44 \text{ cm}^3$$

$$\frac{4\pi (1.5)^3}{3} = 134.05 \text{ cm}^3$$

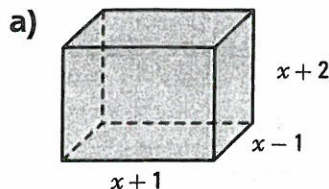
$$134.05 - 65.44 = 68.61 \text{ cm}^3$$

$$134.05 - 32.72 = 101.33 \text{ cm}^3$$

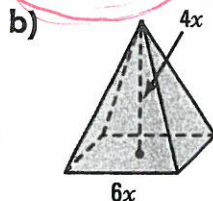
73. The interior and exterior diameters of a metallic ball are 8 cm and 10 cm respectively. What is the ball's mass if the mass of 1 dm^3 of this metal is 3 kg?

$$0.77 \text{ kg}$$

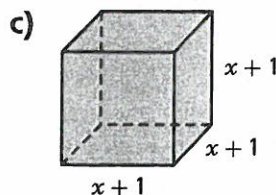
74. Express, using the variable x , the total area and volume of each of the following solids.



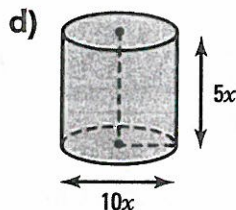
$$V = x^3 + 2x^2 - x - 2$$



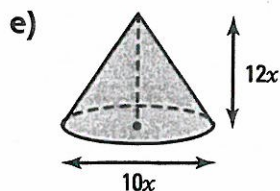
$$V = 48x^3$$



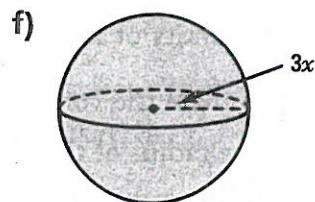
$$V = x^3 + 3x^2 + 3x + 1$$



$$V = 125\pi x^3$$



$$V = 100\pi x^3$$



$$V = 36\pi x^3$$

75. The cube on the right has edges equal to $3x - 2$.

- a) Express, using the variable x ,
1. the total area of the cube

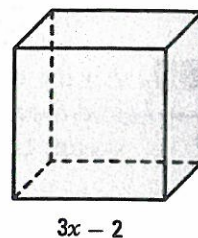
2. the volume of the cube

$$27x^3 - 54x^2 + 36x - 8$$

- b) Calculate, when $x = 2$ cm,
1. the total area of the cube

2. the volume of the cube

$$64 \text{ cm}^3$$



76. The prism on the right has the dimensions: $2x - 1$, $x + 1$ and $x - 1$.

- a) Express, using the variable x ,
1. the total area of the prism

2. the volume of the prism

$$2x^3 - x^2 - 2x + 1$$

- b) Calculate, when $x = 3$ cm,
1. the total area of the prism

2. the volume of the prism

$$40 \text{ cm}^3$$

