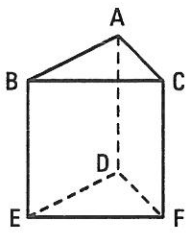
5.3 Prisms – Pyramids – Cylinders

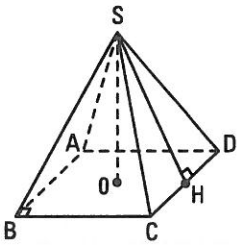
Activity 1 Describing a solid

a) Consider the given right prism.



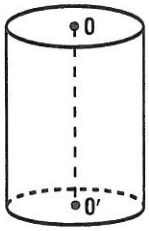
1. Indicate the nature of this prism's bases.
   The bases are triangles.
2. Are the bases parallel and congruent? ***Yes***
3. Indicate the nature of this prism's lateral faces.
   The lateral faces are rectangles.
4. Are the lateral faces congruent and perpendicular to the bases?
   Yes
5. Name one edge whose length is the height of this prism.
   The edge AD, BE or CF.

b) Consider the given right regular pyramid.



1. Indicate the nature of the base. ***A square***
2. Indicate the nature of the lateral faces.
   The lateral faces are isosceles triangles.
3. Are the lateral faces congruent?
   Yes
4. What does the height SH of the triangle SCD represent for the pyramid? ***The slant height***
5. The point O is located at the centre of the base. What does the segment SO represent for the pyramid? ***The height of the pyramid***

c) Consider the given right circular cylinder.



1. Indicate the nature of the bases.
   The bases are discs.
2. Are the bases parallel and congruent? ***Yes***
3. Is the lateral surface plane or rounded?
   Rounded
4. What does the segment OO′ joining the two centres of the bases represent for the cylinder?
   The height of the cylinder

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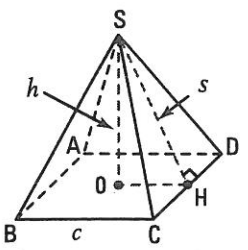
ACTIVITY 2 A remarkable triangle within the pyramid

Consider the triangle SOH in the given right square base pyramid. The point O is located at the centre of the base.



a) What is the nature of the triangle SOH? ***A right triangle***

b) What relation can be deduced between the height h of the pyramid, the side length c of the base and the slant height s? Justify your answer.

Since triangle SOH is a right triangle at O, the Pythagorean Theorem states that

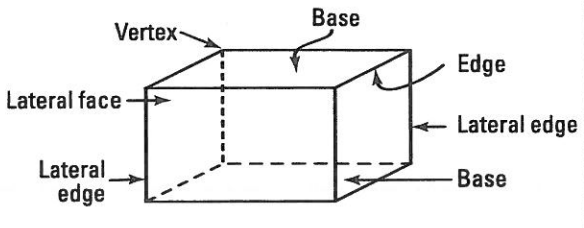
$h^2 + \left(\frac{c}{2}\right)^2 = s^2$ ***or*** $h^2 + \frac{c^2}{4} = s^2$

c) Find the slant height of the pyramid if $c = 6$ cm and $h = 4$ cm.

s = 5 cm

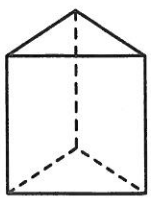
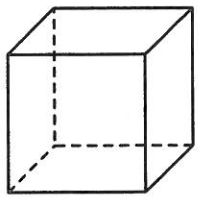
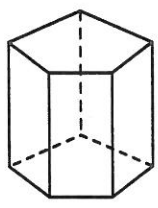
PRISMS

- A **right prism** is a solid limited by
  - **two parallel** and congruent polygons called the prism's **bases**;
  - **rectangular lateral faces** that are perpendicular to the bases.
- Properties of right prisms:
  - The bases are **parallel** and **congruent**.
  - The lateral faces are **perpendicular** to the bases.
  - The lateral edges are parallel, **congruent** and **perpendicular** to the bases.
- The **height** of the prism is the length of one of the lateral edges.



Right rectangular base prism

Ex.:

Right triangular prism	Right square base prism	Right pentagonal base prism
		

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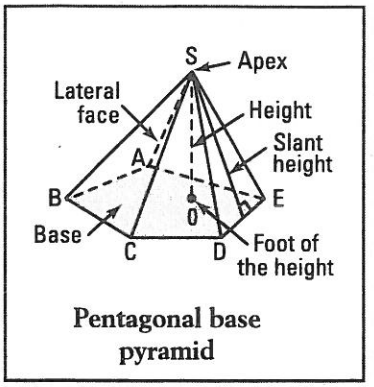
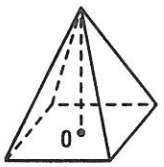
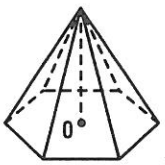
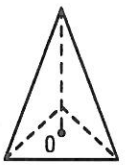
PYRAMIDS

- A **pyramid** is a solid limited by
  - a **polygon** called the **base** of the pyramid;
  - **lateral faces** in the shape of **triangles**.
- The height of each lateral face is called **slant height**.
- A pyramid is **right and regular** when its base is a **regular polygon** and when the **foot** of the height is the **centre** of the base. The lateral faces are then **congruent isosceles triangles**.



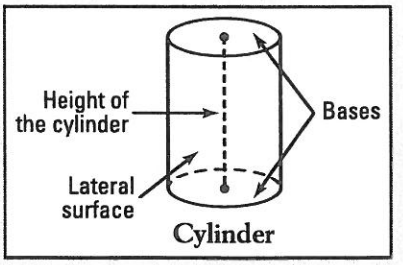
Pentagonal base pyramid

Ex.:

Right square base pyramid	Right regular hexagonal base pyramid	Right regular triangular base pyramid
		

CYLINDERS

- A **right circular cylinder** is limited by
  - **two discs** with the same radius and parallel called the cylinder's **bases**;
  - a curved surface called the **lateral surface** or **cylindrical surface**.
- The radius of the base is called the **cylinder's radius**.
- The segment joining the centres of the bases is the **height** of the cylinder.

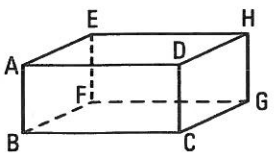


Cylinder

In order to simplify classification, it is generally agreed upon to use the words **prism**, **pyramid** and **cylinder** to represent any right prism, right regular pyramid or right circular cylinder.

1. **a)** Name the solid on the right which is laying on its base.

It is a rectangular base prism.



b) Complete the description.

1. BCGF is ***a base***.
2. A is ***a vertex***.
3. $\overline{AB}$ is ***an edge***.
4. ABCD is ***a face***.

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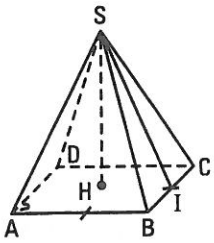
2. a) Name the given right solid.

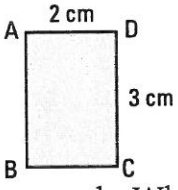
It is a square base pyramid.



b) Complete the description.

1. ABCD is ***the base.***
2. S is ***the apex.***
3. $\overline{SH}$ is ***the pyramid's height.***
4. $\overline{SI}$ is ***the slant height.***

3. a) Explain how to generate a cylinder with a height of 3 cm using the rectangle on the right. What will the cylinder's radius be?



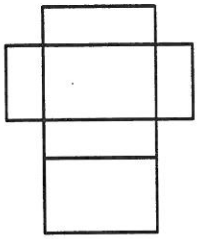
We rotate the rectangle one complete revolution around its length. The radius is 2 cm.

b) Explain how to generate a cylinder with a height of 2 cm using the same rectangle. What will the cylinder's radius be?

We rotate the rectangle one complete revolution around its width. The radius is 3 cm.

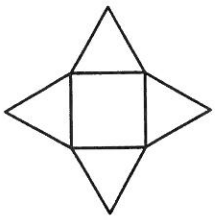
4. The nets of various solids are represented below. Name each solid based on its net.

a)



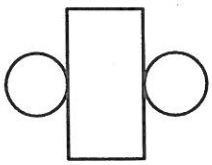
Rectangular base prism

b)



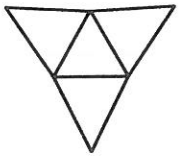
Square base pyramid

c)



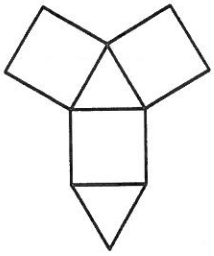
Cylinder

d)



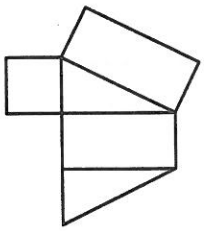
Triangular base pyramid

e)



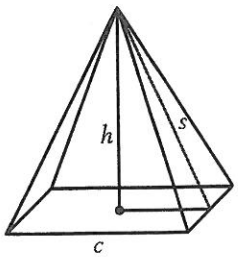
Triangular base prism

f)



Triangular base prism

5. The pyramid on the right with height h and slant height s has a base with side length c. Complete the following table.



h	c	s
8	12	***10***
12	***10***	13
4	6	5

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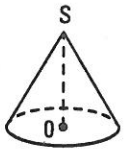
5.4 Cones

Activity 1 Describing a right circular cone



The given right circular cone with apex S is resting on its base, the disc with centre 0.

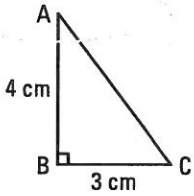
a) What is the lateral surface called? ***The conical surface***

b) Which segment represents the cone's height? ***The segment SO***

c) Let M represent any point on the base's circle.
What is the segment SM called? ***The slant height***

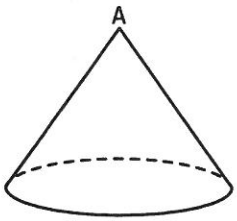
Activity 2 Generating a right circular cone by rotation

The triangle on the right has sides of 3 cm and 4 cm forming a right angle.



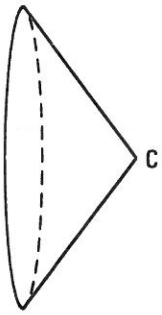
a) A complete revolution around the side AB generates a right circular cone.

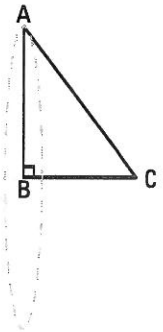
1. Draw the resulting cone.
2. What is the cone's height? ***4 cm***
3. What is the radius of the base? ***3 cm***
4. What is the apex? ***A***



b) A complete revolution around the side BC generates a right circular cone.

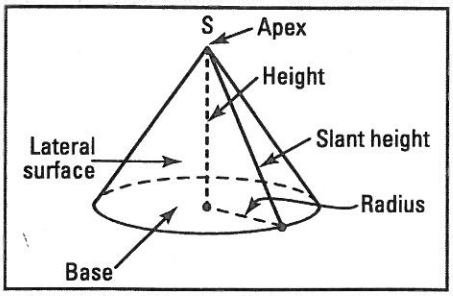
1. Draw the resulting cone.
2. What is the cone's height? ***3 cm***
3. What is the radius of the base? ***4 cm***
4. What is the apex? ***C***





CONES

- A right circular **cone** is a solid limited by
  - a disc called the cone's **base**;
  - a curved surface called **lateral surface** or **conical surface**.
- The radius of the base is called the **radius of the cone**.
- The segment joining the cone's apex to the centre of the base is the **cone's height**.
- Any segment joining the cone's apex to a point on the base's circle is called a **slant height**.



In order to simplify classification, it is generally agreed upon to use the word cone to represent any right circular cone.

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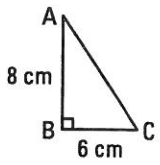
1. a) Explain how to generate a cone with a height of 8 cm using the triangle on the right. What will its radius be?



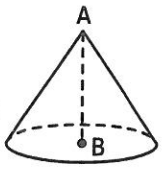
A complete revolution of this triangle around the side AB. r = 6 cm

b) Which point represents

1. the cone's apex? *A* 2. the centre of its base? *B*

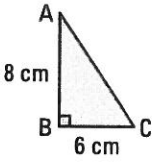
c) Which side of the triangle generates

1. the base of the cone? *Side BC* 2. the conical surface? *The hypotenuse AC*



d) Draw a view of this generated cone.

2. a) Explain how to generate a cone with a height of 6 cm using the same triangle as the preceding question. What will its radius be?



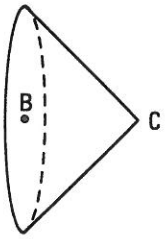
A complete revolution of this triangle around the side BC. r = 8 cm

b) Which point represents

1. the cone's apex? *C* 2. the centre of its base? *B*

c) Which side of the triangle generates

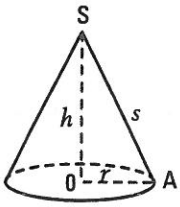
1. the base of the cone? *Side AB* 2. the conical surface? *The hypotenuse AC*



d) Draw a view of this generated cone in perspective.

ACTIVITY 3 A remarkable triangle within the cone

The cone on the right has height h and radius r. The segment SO represents the height of the cone, segment OA represents a radius and segment SA represents a slant height of length s.



a) What is the nature of triangle SOA? *Right in O*

b) What is the relation between the slant height s, the height h and the radius r of the cone? Justify your answer.

$s^2 = h^2 + r^2$ (Pythagorean Theorem applied to triangle SOA)

c) Determine

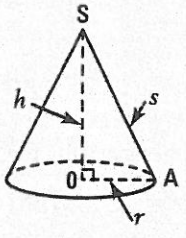
1. s if $h = 4$ cm and $r = 3$ cm. *$s = 5$ cm*
2. h if $s = 6$ cm and $r = 3{,}6$ cm. *$h = 4.8$ cm*
3. r if $s = 7$ cm and $h = 5{,}6$ cm. *$r = 4.2$ cm*

PROPERTIE OF CONES

The cone on the right has radius r, height h and slant height s.

Thus,

$$s^2 = h^2 + r^2$$



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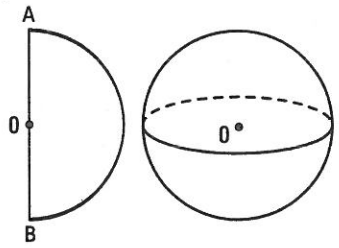
5.5 Sphere

Activity 1 Sphere generated by a rotation

A complete revolution of the semi-disc on the right around the diameter AB will generate a solid.



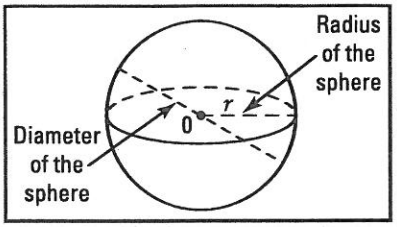
a) What is this solid called? ***A sphere*** Draw it.

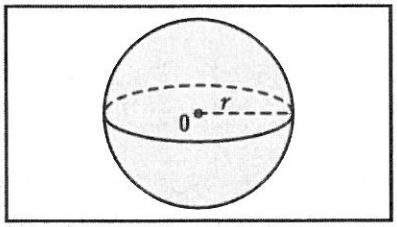
b) Is the surface generated by the semi-circle AB straight or curved? ***Curved***

What is this surface called? ***Spherical surface***

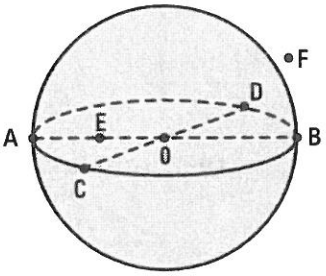
SPHERES

- The spherical surface with centre O and radius r is the set of all points in space located at a distance from the centre equal to r.
  - Any segment joining the centre of the sphere to a point on the spherical surface is a **radius of the sphere**.
  - Any **segment** joining two points on the spherical surface and passing through the centre of the sphere is a diameter of the sphere.
- A sphere with centre O and radius r is the set of all points in space whose distance to the centre is less than or equal to r.





1. Consider the sphere with centre O and a radius of 3 cm.



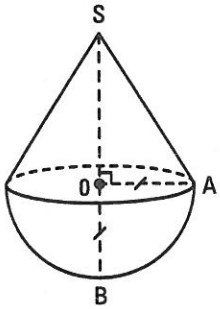
a) Complete.

1. $\overline{OA}$ is ***a radius***.
2. $\overline{AB}$ is ***a diameter***.
3. E is a point ***inside the sphere***.
4. C is a point ***on the sphere***.
5. F is a point ***outside the sphere***.

b) Complete.

1. m$\overline{OC}$ ***= 3 cm***. 2. m$\overline{CD}$ ***= 6 cm***. 3. m$\overline{OE}$ ***< 3 cm***. 4. m$\overline{OF}$ ***> 3 cm***.

2. **a)** Describe with precision the solid on the right.



It is a hemisphere topped by a right circular cone.

b) Complete the description.

1. O ***is the foot of the cone's height and the centre of the hemisphere.***
2. $\overline{SO}$ ***is the height of the cone.***
3. $\overline{OA}$ ***is the radius of the cone and the hemisphere.***
4. $\overline{SA}$ ***is the slant height of the cone.***

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