

## SYSTEM OF 1ST DEGREE EQUATIONS WITH TWO VARIABLES

**Ex.:** Emily wants to rent a car for the weekend. She has the choice of two rental companies. Company A charges \$0.30 per kilometer whereas company B charges a base fee of \$30 plus \$0.20 per kilometer. The rule for calculating the cost  $y$  (in \$) as a function of the number of kilometers  $x$  is given below for each company.

Company A:  $y = 0.30x$     Company B:  $y = 0.20x + 30$ .

Searching for the number  $x$  of kilometers for which the two companies would charge the same amount leads to solving the following system of equations:

$$\begin{cases} y = 0.30x & (A) \\ y = 0.20x + 30 & (B) \end{cases}$$

- Solving a system of equations consists of finding the ordered pair that is a common solution to both equations in the system.

This system can be solved by comparing:

### 1. The table of values

**Company A**

Number of km ( $x$ )	100	200	300	400
Total cost ( $y$ )	30	60	90	120

**Company B**

Number of km ( $x$ )	100	200	300	400
Total cost ( $y$ )	50	70	90	110

Notice that the two companies charge the same amount (\$90) for 300 km.

### 2. The rules

$$\begin{aligned} \text{Solve the equation: } 0.30x &= 0.20x + 30 \\ 0.10x &= 30 \\ x &= 300 \end{aligned}$$

By substituting  $x$  by 300 in one of the equations, you get  $y = 90$ .

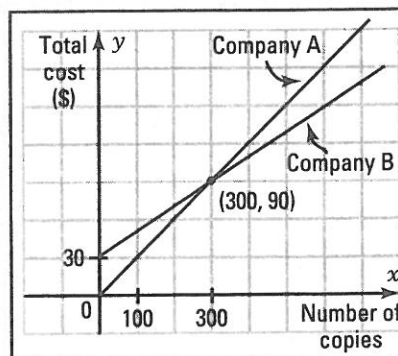
The ordered pair (300, 90) is called **the solution** to the system.

### 3. The Cartesian graphs

- Represent, in the same Cartesian plane, both functions. The point of intersection of the two lines (300, 90) is the solution to the system.
- A more detailed study of the graph reveals that it is more economical to go with company A for a distance less than 300 km and more economical to go with Company B for a distance of more than 300 km.

The solution set  $S$  has only one ordered pair.

We write:  $S = \{(300, 90)\}$ .





Extra practice

Answer Key

1. In each of the following systems, determine the solution.

a)  $\begin{cases} y = 2x + 3 \\ y = -x + 6 \end{cases}$

b)  $\begin{cases} y = 2x + 8 \\ y = x + 5 \end{cases}$

c)  $\begin{cases} y = \frac{1}{2}x \\ y = 2x - 9 \end{cases}$

d)  $\begin{cases} y = 4x + 3 \\ y = 8x + 1 \end{cases}$

e)  $\begin{cases} y = \frac{3}{2}x - 8 \\ y = \frac{1}{4}x - 3 \end{cases}$

f)  $\begin{cases} y = \frac{2}{3}x + \frac{17}{12} \\ y = \frac{1}{4}x + 1 \end{cases}$

2. In each of the following situations

1. identify the variables.
2. translate the situation into a system of two 1st degree equations with two variables.
3. solve the system by comparison and give a complete answer.

must make the equations equal each other

a) An appliance salesman is offered two salary propositions. The first is a base salary of \$300 and a 2% commission on the total amount of his sales. The second is a base salary of \$100 and a 3% commission on the total amount of his sales. For what sales amount will the salesman have the same salary for both salary propositions?

1. let  $x = \text{sales amount}$  2.  $y_1 = 0.02x + 300$  3.  $0.02x + 300 = 0.03x + 100$   
 let  $y = \text{salary}$   $y_2 = 0.03x + 100$   $x = 20000$

b) A travel agency wants to organize a trip to the Laurentians. It has the choice of the two following transportation companies: company A charges a base fee of \$80 plus \$10 per passenger and company B charges a base fee of \$120 plus \$8 per passenger. For what number of passengers will the transportation costs be the same for both companies?

1. let  $x = \text{\# of passengers}$  2.  $y_1 = 10x + 80$  3.  $10x + 80 = 8x + 120$   
 let  $y = \text{cost of transportation}$   $y_2 = 8x + 120$   $x = 20$

3. Mr. and Mrs. Smith want to go on a cruise of at least 5 days for their wedding anniversary. The graph on the right illustrates the two options available to them.

a) Translate this situation by a system of equations.

$y_1 = 40x + 1000$   
 $y_2 = 60x + 600$

b) How many days must their cruise last for

1. the first option to be more economical?

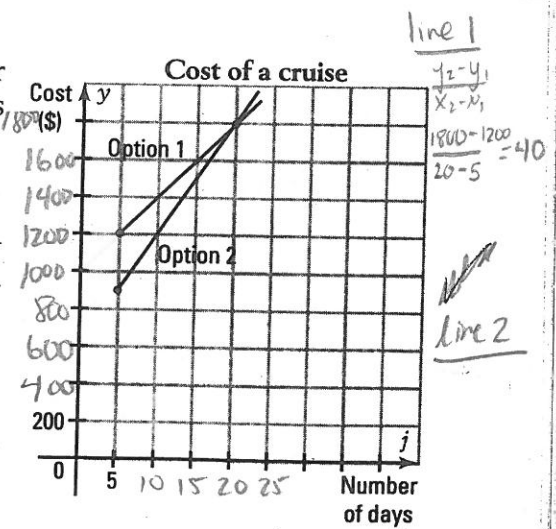
20 < days and up

2. the second option to be more economical?

> 20 days and down

3. the cost to be the same for both options?

20 days (look at graph)



③ line 1 (option 1)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 1200}{20 - 5} = 40$$

$y_1 = 40x + b$   
plug in coordinate to find b

$$1200 = 40(5) + b$$

$$1000 = b$$

a)

$$\boxed{y_1 = 40x + 1000}$$

line 2 (option 2)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1800 - 900}{20 - 5} = \frac{900}{15} = 60$$

$$y_2 = 60x + b$$

plug in coordinate to find b

$$900 = 60(5) + b$$

$$600 = b$$

a)

$$\boxed{y_2 = 60x + 600}$$

b)

1) Proof @ 25 days

option 1:  $y_1 = 40(25) + 1000$  ← option 1 is less expensive  
 $y_1 = 2000$

$$\text{option 2: } y_2 = 60x + 600$$

2) Proof @ 5 days

$$\text{option 1: } 1200$$

$$\text{option 2: } 900$$

← option 2 is less expensive.

$$3) 40x + 1000 = 60x + 600$$

$$\frac{-20x}{-20} = \frac{-400}{-20}$$

$$x = 20$$



10.

A video club offers its members two membership options.

**Option A:** \$10 annual membership fee plus \$2.50 per rented movie.

**Option B:** No annual membership fee and \$3 per rented movie.

Represent this situation in the Cartesian plane. What option would you choose if you had to become a member for one year?

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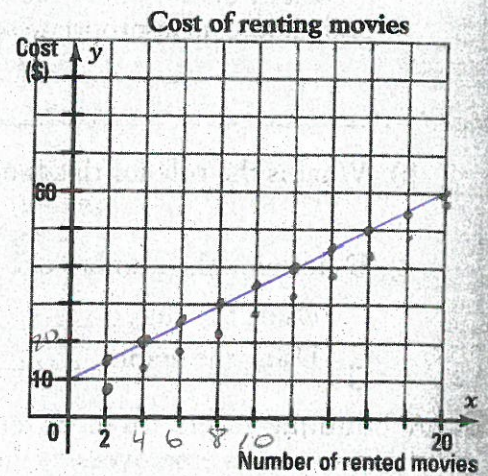
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Option A :  $y_1 = 2.50x + 10$

Option B :  $y_2 = 3x$

\* it depends on the # of movies rented.

Justify your answer.

